Paper Code: RCA103 **Roll No.** | | | | | | | | | | |

B. Tech.
FIRST SEMESTER THEORY EXAMINATION, 2016-17
DISCRETE MATHEMATICS

[Time: 3 Hours] **[Total Marks: 70]**

Note: Attempt ALL questions. All question carry equal marks.

1. Attempt any ***four*** parts of the following: [3.5x4 = 14]

(a) Define countable and uncountable sets and give their examples.

(b) Let S_k={ …, -1, 0, 1,...}.Find $\bigcup_{k=1}^{n} S_k$and $\bigcup_{k=1}^{\infty} S_k$.

(c) Prove that if R is an antisymmetric relation, then R^{-1} is also antisymmetric.

(d) Let the function $f: R\rightarrow R$ where $f(x)=x^2$. Find

$$f^{-1}(-\infty \leq x \leq 0)$$

(e) Use mathematical induction to show that n^{th} Fibonacci number $f_n< (13/8)^n$ for all positive integers $n \geq 1$.

(f) Find the least value of n for which the equality is true and then prove that $(1+ n^2) < 2^n$, where n is the natural number.

2. Attempt any ***four*** parts of the following: [3.5x4 = 14]

(a) Define partial order sets with example.

(b) What is Hasse diagram? Consider a set S={a, b, c}. Draw the Hasse diagram for the partial order set (ρ(S), ⊆), where ρ(S) is a power set of S.

(c) Define a lattice with example and describe its properties.

(d) Find all sublattices of the lattice $(S_k, \leq)$ for $k = 12$, where $a \leq b$ means a divides b for any $a, b \in S_k$.

(e) Find the complements of every element of the lattice $(S_k, \leq)$ for $k = 75$, where $a \leq b$ means a divides b for any $a, b \in S_k$.

(f) Define the morphism of a lattice with example.

3. Attempt any ***two*** parts of the following: [7x2 = 14]

(a) $f(x, y, z) = xy' + xyz' + x'yz'$, then show that

(i) $f(x, y, z)+ xz' = f(x, y, z)$

(ii) $f(x, y, z) + z' \neq f(x, y, z)$

(b) Simplify the following Boolean function using K-map

$$f(w, x, y, z) = \prod(0, 1, 2, 3, 4, 6, 12)$$

(c) (i) How are sequential circuits different from combinational circuits?

(ii) Show the logic diagram of a clocked D flip-flop.

4. Attempt any ***two*** parts of the following: [7x2 = 14]

(a)Test the equivalence of the following compound statements

(i) $P \to (Q \wedge R)$ and $(P \to Q) \wedge (P \to R)$

(ii) $P \to (Q \to R)$ and $(P \wedge Q) \to R$

(b) Determine a suitable conclusion for each of the following premises.

(i) $P \to \neg Q$, $R \to P$ and Q

(ii) $P \to \neg Q$, $\neg P \to Q$ and $\neg P$

(c) (i) Describe the rules of inference for predicate logic

(ii) Negate the following predicate formula

$$(\forall x)P(x) \to (\exists x)\big(Q(x) \wedge S(x)\big)$$

5. Attempt any ***two*** parts of the following: [7x2 = 14]

(a) (i) Compute the asymptotic order of the following function

$$f(n) = 5.2^n + n \log n$$

(ii) Show that

$$\sum_{i=1}^{n} \frac{1}{i} = O(\log n)$$

(b) Solve the following recurrences for T(1) = O(1).

(i) $T(n) = 2\,T(n/2) + 7n$, where $n \geq 2$ and a power of 2.

(ii) $T(n) = T(n/2) + n \log n$, where $n \geq 2$ and a power of 2.

(c) (i) Determine the number of strings possible of lowercase letters of length five or less.

(ii) There are 4 men and 6 women. Each man marries one of the women. In how many ways can this be done?