Paper Code: RAS -103 **Roll No.**

B. TECH.
FIRST SEMESTER EXAMINATION, 2016-17
ENGINEERING MATHEMATICS -I

[Time: 3 hrs.] [Max. Marks: 70]

Note: Attempt ***ALL*** questions. Assume suitable data, if required. All question carry equal marks.

1. *Attempt any* ***two*** *parts of the following:–* **(7x2=14)**

(a) If $\log y = tan^{-1}x$ then show that
$(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$
hence find y_3, y_4 & y_5 at $x = 0$

(b) Evaluate $\left(x^2\ \frac{\partial^2 u}{\partial x^2} + 2xy\ \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}\right)$
If $u = sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}}\right)$

(c) If $w = sin^{-1}u$ and $u = \frac{x^2+y^2+z^2}{x+y+z}$ then show that $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\ \frac{\partial w}{\partial z} = \tan w$

2. *Attempt any* ***two*** *parts of the following:–* **(7x2=14)**

(a) If $u = \left(\frac{x+y}{z}\right), v = \left(\frac{y+z}{x}\right)$ & $w = \left[\frac{y(x+y+z)}{xz}\right]$
show that u, v, w are dependent and find the relation between them

(b) Express the function $f(x,y) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's series expansion about the point $(1,2)$

(c) Find the maximum and minimum distance of the point (a,b,c) from the sphere $x^2 + y^2 + z^2 = r^2$

OR

(c) Trace the curve $y^2 = x^2\left(\frac{a+x}{a-x}\right)$

3. *Attempt any* ***two*** *parts of the following:–* **(7x2=14)**

(a) If the eigen vectors of matrix $\begin{bmatrix}1 & 2\\ 0 & 2\end{bmatrix}$ are writen in the form $\begin{bmatrix}1\\ a\end{bmatrix}$ & $\begin{bmatrix}1\\ b\end{bmatrix}$. What is the value of $(a+b)$

(b) *Reduce the matrix to normal form and hence find the rank of the following matrix*

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & 7 \end{bmatrix}$$

(c) *Use Cayley − Hamilton theorem to find* $|A^{-2}|$ *for the matrix*

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

4. *Attempt any* ***two*** *parts of the following*: − **(7x2=14)**

(a) *Changing the order of integration in the double integral* $I = \int_0^8 \int_{\frac{x}{4}}^{2} f(x,y)\ dy\, dx$

leads to $I = \int_r^s \int_p^q f(x,y)\ dx\, dy$ *, what is the value of q*

(b) *Using triple integral evaluate the volume enclosed by surface*

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1$$

(c) *Show that* $\dfrac{\Gamma\frac{1}{5}\,\Gamma\frac{7}{10}}{\Gamma\frac{2}{5}} = 2^{3/5}\sqrt{\pi}$

5. *Attempt any* ***two*** *parts of the following*: − **(7x2=14)**

(a) *Prove the following vector identities*

$(i)\, div\,(\phi\, \vec{f}\,) = \phi\ div\, \vec{f} + \vec{f}.\, grad\ \phi$

$(ii)\, curl\,(\phi\, \vec{f}\,) = grad\ \phi \times \vec{f} + \phi\ curl\ \vec{f}$

when ϕ *is scaler and* $\vec{f}$ *is vector*

(b) *Verify Green's theorem for* $\int_C (x^2 - 2xy)dx + (x^2 y + 3)\ dy$ *around the curve*

$y^2 = 8x$ & $x = 2, y = 0$

(c) *Verify the Gauss divergence theorem for vector* $\vec{F} = xy\hat{\imath} + z^2\hat{\jmath} + 2yz\,\hat{k}$

bounded by tetrahedron $x = 0, y = 0, z = 0$ *and the plane* $x + y + z = 1$

OR

Evalate the surface integral $\iint_S [x\ dy\ dz + y\ dx\ dz + z\ dx\ dy]$ *where S is portion of the plane* $x + 2y + 3z = 6$ *which lies in the first octant.*