Paper Code: OE-038	**Roll No.**

B.Tech.
THIRD SEMESTER EXAMINATION, 2016-17
DISCRETE MATHEMATICS

[Time:3 Hours] [Total Marks: 100]

Note: Attempt ***ALL*** questions. Assume suitable data, if required. All question carry equal marks.

1. Attempt any *FOUR* parts of the following :- **(5x4=20)**

(a) If R and S are equivalence relations on the set A, show that the following are equivalence relations :-
(i) $R \cap S$ (ii) $R \cup S$

(b) Let X = {a,b,c}. Define f : X → X such that f = { (a,b), (b,a), (c,c)}. Find :
(i) f^{-1} *(ii)* f^2 *(iii)* f^3

(c) Let S be the set of all points in a plain. Let R be a relation such that for any two points, a and b ; (a, b) ∈ R if b is within two centimetre from a, show that R is an equivalence relation.

(d) Show that for any two sets A and B
$A-(A\cap B)=A-B$, without Venn diagram.

(e) Let f, g, h ∈ R be defined as
$f(x)=x+2,\ g(x)=x-2,\ h(x)=3x \ \ \forall\ \ x\in R$
Find $gof,\ hof,\ fohog$.

(f) Let A, B, C be two subsets of U. Given that $A\cap B=A\cap C$, is it necessary that B = C ? Justify your answer.

2. Attempt any *TWO* parts of the following :- **(10x2=20)**

(a) Find out whether the following propositions are tautologies :
(i) $p\wedge(q\wedge r)\Leftrightarrow(p\wedge q)\wedge r$
(ii) $(p\wedge q)\Rightarrow(p\Rightarrow q)$

(b) Show that the following pair of propositions are logically equivalent :
$$(p\vee q)\Rightarrow r \quad and \quad (p\Rightarrow r)\wedge(q\Rightarrow r)$$
Use truth table as well as algebra of proposition to show.

(c) Prove the validity of the following argument :
" If I get the job and work hard, then I will get promoted.
If I get promoted, then I will be happy.
I will not be happy.
Therefore, either I will not get the job or I will not work hard."

3. Attempt any *TWO* parts of the following :- **(10x2=20)**

(a) (i) How many number greater than one million can be formed with the digits 4, 6, 6, 0, 3, 6, 3 ?
(ii) How many different choices can be made of selections out of 15 maths, 10 numerical analysis and 12 operational research books when at least one book is to be selected ?

(b) Solve the following recurrence relation
$$a_r+5a_{r-1}+6a_{r-2}=3r^2-2r+1$$
With the initial condition $a_0 = 1$ and $a_1 = 2$.

(c) Solve the following recurrence relation, using Generating Function

$$a_r - 9a_{r-1} + 26a_{r-2} - 24a_{r-3} = 0$$

For $r \geq 3$, with the initial condition $a_0 = 0$, $a_1 = 1$ and $a_2 = 10$.

4. Attempt any *FOUR* parts of the following :- **(5x4=20)**

(a) Let G be the set of all non- zero real numbers and let $a * b = \frac{ab}{2}$.

Show that (G, $*$) is an abelian group.

(b) Show that the four matrices

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

form a group with binary operation multiplication of matrices.

(c) Show that the set M of all 2x2 matrices over integers form a ring under matrix addition and multiplication.

(d) Show that (R, +, **.**) is a field where R is a set of all real numbers.

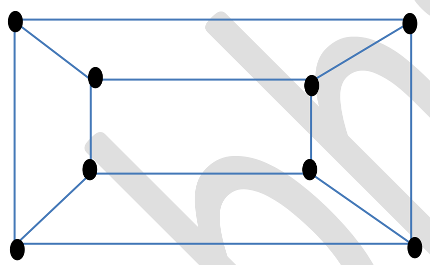
(e) Show that the set $\{1, \omega, \omega^2\}$, where ω is the cube root of unity, form a finite multiplicative abelian group.

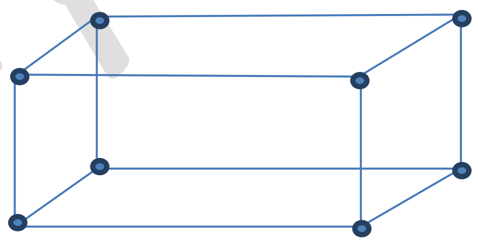
(f) Prove that the set P_3 of all permutations on X = { a, b, c } is a finite group with respect to product of mappings as the operation.

5. Attempt any *FOUR* parts of the following :- **(5x4=20)**

(a) Prove that the number of vertices of odd degree in a connected graph is always even.

(b) Define isomorphism of graphs. Find out whether the following graphs are isomorphic or not also explain your answer.





(c) Discuss the Travelling Sales Person Problem.

(d) Define tree with example. If G is a tree with n vertices then prove that it has exactly n-1 edges.

(e) Define Binary Tree with one example. Find the number of pendant vertices in a binary tree of n vertices.

(f) Define chromatic number of a graph. Find the chromatic number of the following graph.