Paper Code: MCA114	**Roll No.**									

MCA
(SEM I) CARRY OVER EXAMINATION 2016-17
DISCRETE MATHEMATICS

[Max. Marks: 100] **[Time: 3.00 hr]**

Note: Attempt all questions. All questions carry equal marks.

1. Attempt any **TWO** parts of the following: [10x2=20]
   a) (i) Let S = {ϕ, 1, 2, 3}. Find ρ(S) where ρ(S) is a power set of S.
   (ii) Let S and T be sets. Then prove that S ∩ (T – S) = ϕ.
   b) (i) State the condition for which a relation *R* on a set of integers is not transitive.
   (ii) Prove that if R is an symmetric relation, then R ∩ R^{-1} = R.
   (c) Show that x^{n-1}– 1is divisible by $x - 1$, for all $n \geq 0$.

2. Attempt any **TWO** parts of the following: [10x2=20]
   (a) (i) Let *G* be a group and let $x^2 = 1$, for all $x \in G$. Show that *G* is Abelian.
   (ii) Prove that a group *G* is Abelian if and only if $(xy)^{-1} = x^{-1}y^{-1}$, for all $x, y \in G$.
   (b) Prove that the intersection of two subgroups of a group *G* is also a subgroup of *G*.
   (c) Define a field. How does a field differ from a ring?

3. Attempt any **TWO** parts of the following: [10x2=20]
   (a) What is Hasse diagram? Consider a set S={1, 2, 3}. Draw the Hasse diagram for the partial order set (ρ(S), ⊆), where ρ(S) is a power set of S.
   (b) Consider the Boolean function $f(x,y,z) = xyz + x'y'z + xy'z$
      (i) Find the truth table for *f*.
      (ii) Simplify the expression using Boolean algebra and identities.
   (c) What is the basic element of sequential circuit? Give an example of sequential circuit.

4. Attempt any **TWO** parts of the following: [10x2=20]
   (a) What is a tautology? Prove that $(\sim Q \wedge (P \rightarrow Q)) \rightarrow \sim Q$ is a tautology.

(b) (i) Show that the following argument is valid.

$$P \ /\therefore \sim(Q \wedge \sim Q)$$

(ii) Determine a suitable conclusion drawn from the following premises.

$$P \rightarrow \sim Q, R \rightarrow P \text{and} Q.$$

(c) (i) Illustrate the rules of inference for predicate logic.

(ii) Negate the following predicate formula

$$(\exists x)(\forall y)(P(y) \rightarrow Q(x))$$

5. Attempt any **TWO** parts of the following: [10x2=20]

(a) (i) Define a bipartite graph with example.

(ii) Explain Eulerian graph with example.

(b) Solve the following recurrence for *T(1) = O(1)*.

$$T(n) = 2T\left(\frac{n}{2}\right) + 7n \text{ where } n \geq 2 \text{ and a power of 2.}$$

(c) (i) Ten people split up into five groups of two each. In how many ways can this be done?

(ii) Determine the number of bit strings of length 10 that either begin with three 0's or end with two 1's.