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Paper Code: CS-303	**Roll No.**										

B.Tech.
(SEM III) ODD SEMESTER THEORY EXAMINATION, 2016-17
COMPTER BASED NUMERICAL AND STATISTICAL TECHNIQUES

[Time: 2 Hours] [**Max. Marks: 50**]

Note:
(i) *Attempt **ALL** questions.*
(ii) *Notations/ Symbols/ Abbreviations used have usual meaning.*

Q.1. Attempt any **FOUR** parts of the following:- (**3.5x4=14)**

a) Explain the following terms.
Rate of Convergence,
Illconditioned system of equations,
Machine Epsilon

b) Show at least two scenarios through graphical sketch when choice of initial guess in Newton Raphson method may lead to divergence or endless cycle.

c) Determine the vaue of **p** and **q** so that the order of iterative method

$$x_{n+1} = px_n + qa/x_n^2$$

for computing root of the equation $x^3 - a = 0$ is as high as possible.

d) Derive the expression for Aitken acceleration of improving linear convergence of an iterative method.

e) Perform four iterations of the Newton Raphson method with initial guess of **3** to find the approximate value of cube root of **23**.

f) Using Sturm theorem, determine the number of real roots of the given polynomial in the interval **[-3, 3]** with their multiplicity.

$$x^4 - 8x^2 + 1 = 0$$

Q.2. Attempt any **TWO** parts of the following- **(6x2=12)**

a) Show that polynomials $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = x^2 - 1/3$ are orthogonal polynomials over the interval **[-1, 1]** with respect to weight function **W(x) =1.** Use these polynomials to obtain normal equations for second degree approximation of $f(x) = x^4$ on **[-1, 1]** according to least squares principle.

b) Obtain the natural cubic spline interpolating polynomial valid in the interval **[2, 3]** for the given function **f(x)**.

x_i	1	2	3	4
$f(x_i)$	3	9	27	81

c) Determine the step size **h** that can be used to tabulate the value of e^x at equispaced pints in the interval **[0, 1]** so that error in the quadratic interpolation to the **f(x)** is less than **0.0005**.

Q.3. Attempt any **TWO** parts of the following: **(6x2=12)**

a) Given the following values of **f(x) = log x**, find the approximate values of **df(x)/dx** at **2.2** using quadratic interpolation and obtain the upper bound on the error.

f (2.0) = 0.69315,
f (2.2) = 0.78846,
f (2.6)= 0.95551

b) Write Simpson's 1/3 method of integration and derive the local and global truncation error term for the method.

c) Evaluate the following integral using Guass-Legendre 3-point integration method.

$$\mathbf{I = \int_0^2 dx / (x^2+2x+10)}$$

Q.4. Attempt any **TWO** parts of the following: **(6x2=12)**

a) Find the optimal relaxation parameter for the Successive Over Relaxation (SOR) iteration scheme for solving the given system of simultaneous equations.

$$\mathbf{4x - y = 3}$$
$$\mathbf{-x + 4y - z = 2}$$
$$\mathbf{-y + 4z = 3}$$

Also, determine the rate of convergence of the method.

b) Using fourth order Runge-Kutta method, obtain numerical solution of following differential equation at **x = 0.4**

$$\mathbf{dy/dx = (y+x)/(y-x);\ y(0) = 1}$$

Assume step size **h = 0.2.**

c) Following table shows the observed and expected frequencies in tossing a dice 120 times. Test the hypotheses that dice is fair, using significance level of 0.05.

Face	**1**	**2**	**3**	**4**	**5**	**6**
Observed Frequency	25	17	15	23	24	16
Expected Frequency	20	20	20	20	20	20

Given that for 5 degrees of freedom, value of chi-square (χ^2) at 0.95 and at 0.05 are 11.1 and 1.15 respectively.