Paper Code: CS-302	**Roll No.**										

B.TECH
(III SEM) ODD SEMESTER EXAM 2016-17
DISCRETE STRUCTURES AND GRAPH THEORY

[Time: 3 hours] **[Max. Marks: 100]**

Note- Attempt all question. All question carry equal marks.

Q1. Attempt any **TWO** parts of the following:- **[10 X 2 = 20]**

(a) Out of 250 candidates who failed in an examination, it was revealed that 128 failed in Mathematics, 87 in Physics and 134 in aggregate. 31 failed in Mathematics and Physics, 54 failed in the aggregate and in Mathematics, 30 failed in the aggregate and in Physics. Find how many candidate failed:
(i) in all three subjects; (ii) in Mathematics but not in Physics; (iii) in the aggregate but not in Mathematics; (iv) in Physics but not in the aggregate or in Mathematics; (v) in the aggregate or in Mathematics but not in Physics.

(b) Let $f{:}X\rightarrow Y$ be an everywhere defined invertible function and A and B be arbitrary non-empty subsets of Y. Show that
(i) $f^{-1}(AUB) = f^{-1}(A)\ U\ f^{-1}(B)$
(ii) $f^{-1}(A\cap B) = f^{-1}(A) \cap f^{-1}(B)$

(c) Use mathematical induction to show that :

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{n}} > 2(\sqrt{n+1}\ -1) \text{ for all integers } n \geq 1.$$

Q2. Attempt any **TWO** parts of the following:- **[10 X 2 = 20]**

(a) Let G be a set of all non-zero real numbers and let x*y = xy/2. Then show that (G,*) is an Abelian group. Prove that the set {0,1,2,3,4} is a finite abelian group of order 5 under addition modulo 5 as composition.

(b) Prove that the intersection of two subrings of a ring is a subring R. If in a ring R with unity, $(x\ y)^2 = x^2y^2$ for all x, y ϵ R, then prove that R is commutative.

(c) Let R be a commutative ring with unity and M be an ideal of R. Show that M is a maximal ideal if and only if R/M is a field.

Q3. Attempt any **FOUR** parts of the following:- **[5X 4 = 20]**

(a) Consider a set S={1,2,3,4,5,6,10,12,15,20,30,60} of all divisors of 60. Draw the Hass diagram when partially ordered by divisibility.

(b) Optimize the Boolean function f (w, x, y, z) =$\sum$ (0,1,2,5,8,9,10) and represent it in the sum-of-product and the product-of-sums forms.

(c) Prove that a lattice (L,≤) is modular if and only if (a Λ b)V(a Λ c)=a Λ (b V(a Λ c)) for all a, b, c ϵ L.

(d) Consider the poset A=({1,2,3,4,6,9,12,18,36}, 1) find the greatest lower bound and least upper bound of the sets {6,18} and {4,6,9}.
(e) Show that every chain is a distributive lattice.
(f) Explain the Full Adder and Half Subtractor.

Q4. Attempt any **TWO** parts of the following:- **[10 X 2 = 20]**

(a) Consider the statement, ‘English team loses whenever it rains’. State the contrapositive, converse and inverse statements of the above conditional statement.
(b) Show that the statements ‘No Mortal is perfect’ and ‘All humans are mortal’ imply the conclusion ‘No human is perfect’.
(c) Show that the following argument is valid:

$$(\forall \boldsymbol{x})\ (\boldsymbol{P}(\boldsymbol{x}) \rightarrow (\boldsymbol{Q}(\boldsymbol{x}) \land \boldsymbol{R}(\boldsymbol{x})))\ \textbf{and}\ (\forall \boldsymbol{x})(\boldsymbol{P}(\boldsymbol{x}) \land \boldsymbol{S}(\boldsymbol{x}))\ \ /\therefore\ (\forall \boldsymbol{x})\ (\boldsymbol{R}(\boldsymbol{x}) \land \boldsymbol{S}(\boldsymbol{x})).$$

Q5. Attempt any **TWO** parts of the following:- **[10 X 2 = 20]**

(a) Represent the expression as a binary tree and write the prefix and postfix forms of the expression: **A * B - C ↑ D +E / F.** Prove that the maximum number of vertices in a binary tree of depth d is 2^d -1 where d ≥ 1.
(b) Prove that if a connected planar graph G has n vertices, e edges and r region, then n - e + r = 2. Show that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.
(c) Solve the recurrence relation of the following:

$$\mathbf{Y_{n+2} - Y_{n+1} - 2Y_n = n^2}$$