

(The paper code and roll No. to be filled in your answer book)

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B TECH
(SEM I) CARRY OVER EXAMINATION 2016-17
ENGINEERING MATHEMATICS

TIME: 3 Hours**Total Marks: 100***Note: Attempt all questions. All questions carry equal marks.***Q1. Attempt any two parts of the following:****[10x2=20]**(a) If $Y = \sin(m \sin^{-1} x)$, prove that

$$(1 - x^2)y_{n-2} - (2n - 1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

Also find $y_{n/0}$ (b) If $w = \sqrt{x^2 + y^2 + z^2}$, $x = u \cos v$, $y = u \sin v$ and $z = uv$ then prove that

$$u \frac{\partial w}{\partial u} - v \frac{\partial w}{\partial v} = \frac{u}{\sqrt{1 + v^2}}$$

(c) Trace the curve $3ay^2 = x(x - a)^2$ **Q2. Attempt any two parts of the following:****[10x2=20]**(a) If $u = x(1 - r^2)^{-1/2}$, $v = y(1 - r^2)^{-1/2}$, $w = z(1 - r^2)^{-1/2}$ then show that

$$\frac{\partial(uvw)}{\partial(xyz)} = (1 - r^2)^{-5/2} \text{ where } r^2 = x^2 + y^2 + z^2$$

(b) Expand $f(xy) = e^{x \tan^{-1} y}$ about point (1,1) up to second degree term.(c) Find the maximum value of u where $u = \sin x \sin y \sin(x+y)$ **Q3. Attempt any two parts of the following:****[10x2=20]**(a) Find the rank of matrix by reducing to echelon form $A =$

$$A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$$

(b) For what value of λ the system of equations

$$x+y+z=1,$$

$$x+2y+4z=\lambda,$$

$$x+4y+10z=\lambda^2$$

have a solution and solve them in each case.

(c) Using Caylay's Hamilton theorem, find the inverse of the matrix A^{-1} when

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

Q4. Attempt any two parts of the following:

[10x2=20]

(a) Evaluate the following gamma function: $\frac{\Gamma(n + \frac{1}{2})}{\Gamma(n+1)}$

(b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ and hence show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

(c) Evaluate the triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (xyz) dx dy dz$.

Q5. Attempt any two parts of the following:

[10x2=20]

(a) Show that $\text{curl} \left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5} (\vec{a}, \vec{r})$

(b) Verify the Gauss divergence theorem $\int_c \left[(x^3 - yz)\hat{i} - 2x^2 y\hat{j} + 2z\hat{k} \right] \hat{n} ds$ where s denotes surface of cube bounded by plane $x=0, x=a; y=0, y=a; z=0, z=a$.

(c) Evaluate the surface integral $\int_s [x dy dz + y dz dx + z dx dy]$ where s is the portion of the plane $x + 2y + 3z = 6$ which lies in first octant.