**MME-101 / STR-11** **Roll No.** | | | | | | | | | | |

**M. TECH.**

**FIRST SEMESTER EXAMINATION, 2015-16**

**ADVANCED MATHEMATICS AND NUMERICAL METHODS**

*Time: 3 Hours* *Total Marks: 100*

**Note:** 1. Attempt ***ALL*** questions.

2. Assume suitable data, if required.

**1.** Attempt any ***Two*** parts of the following:- **(10x2=20)**

a) Using Muller's method, find a root of the following equation

$$x^3 + 2x^2 + 19x - 29 = 0$$

b) Derive Newton Raphson method to find a root of the equation $f(x) = 0$. Find the order of convergence of this method.

c) Using Chebyshev method, find a real root of the equation

$$x^4 - x - 10 = 0$$

**2.** Attempt any ***Two*** parts of the following:- **(10x2=20)**

a) Solve the following system of linear algebraic equation by Gauss- Seidel method correct up to five decimal places.

$$19x + 11y + 5z = 37$$

$$13x + 23y + 7z = 41$$

$$29x + 11y + 43z = 103$$

b) Solve the following system of equation by LU decomposition method.

$$x + 2y + 3z = 5$$

$$2x + 8y + 22z = 6$$

$$3x + 22y + 82z = -10$$

c) Describe the Cholesky $LL^T$ -factorization method. Hence, solve the following system of equations

$$5x + 11y + 19z = 29$$

$$11x + 19y + 13z = 51$$

$19x + 13y + 31z = 41$

3. Attempt any ***Two*** parts of the following: - **(10x2=20)**

a) The following table gives the values of *y* which is a polynomial of degree five. It is known that there is error in *f(4)*. Correct the error.

| *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| *y = f(x)* | 1 | 2 | 33 | 254 | 1035 | 3127 | 7777 |

b) Using the Langarage method, find the interpolatory polynomial for the following data:

| *x* | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| *f(x)* | 1 | 14 | 15 | 17 | 5 | 11 |

c) Find the cubic spline interpolation for the following data:

| *x* | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *f(x)* | 1 | 0 | 1 | 0 | 1 |

4. Attempt any ***Two*** parts of the following: - **(10x2=20)**

(a) Evaluate the following using 1/3 -Simpson's rule.

$$\int_0^6 \frac{dx}{1+x3}$$

Also find the error in 1/3 -Simpson's rule.

(b)Use 3/8 Simpson's method to compute the following, correct up to four decimal places.

$$\int_0^1 \frac{dx}{1+x3}$$

c) Find *f'(4)* from the following data.

| *x* | 3 | 5 | 11 | 27 | 34 |
|---|---|---|---|---|---|
| *f(x)* | -13 | 23 | 899 | 17315 | 35606 |

5. Attempt any ***Two*** parts of the following: - **(10x2=20)**

a) Solve the following by Euler's modified method. Perform five iterations.

$$\frac{dy}{dx} = y + x$$

$$y\,(o) = 1$$

b) Apply Runge-Kutta 4th order method to find the approximate value of y for x = 0.6 in step length of 0.2, if

$$\frac{dy}{dx} = x + y2$$

*given that y (o) =1*

c) Solve $\left(\frac{\partial 2u}{\partial x2}\right) + \left(\frac{\partial 2u}{\partial y2}\right) = 0$ over the square region of side 4 units, satisfying the following boundary conditions. (Choose h = 1 unit)

$u(x,0) = 3x \quad for \quad 0 \leq x \leq 4$

$u(x,4) = x^2 \quad for \quad 0 \leq x \leq 4$

$u(0,y) = 0 \quad for \quad 0 \leq y \leq 4$

$u(4,y) = 12+y \quad for \quad 0 \leq y \leq 4$