Paper Code: OE-048 **Roll No.** | | | | | | | | | | |

B.Tech.
(SEM IV) EVEN SEMESTER EXAMINATION, 2015-16
DISCRETE MATHEMATICS

[Time: 3 hrs.] **[Max. Marks: 100]**

Note:-Attempt all questions. All questions carry equal marks.

1. Attempt any two parts of the following:- **[10x2=20]**

(a) Define an equivalence relation. Let S be the set of ordered pairs of positive integer and define R so that $(x_1, x_2)R(y_1, y_2)$ means that $x_1 + y_2 = y_1 + x_2$. Show that R is an equivalence relation.

(b) In a canteen, out of 123 students, 42 students buy ice-cream,36 buy buns and 10 buy cakes, 15 students buy ice-cream and buns, 10 buy ice-cream and cakes, 4 buy cakes and buns but not ice-cream and 11 ice-cream and buns but not cakes. Draw Venn-diagram to illustrate the above information and find
(i) How many students buy nothing at all?
(ii) How many students buy at least two items?

(c) Prove by method of mathematical induction $\frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for every positive n.

Define a mapping $g: Z \to Z$ as $g(x) = x - 2$. Show that g is one to one and onto.

2. Attempt any two parts of the following:- **[10x2=20]**

(a) Solve the recurrence relation given below
$a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$, given that $a_0 = 0$, $a_1 = 1$

(b) Use Generating function to solve the recurrence relation
$a_n - 9a_{n-1} + 20a_{n-2} = 0$, $a_0 = -3, a_1 = -10$

(c) (i) If $1 \leq r \leq n$, prove that ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$
(ii) Find the minimum number of students in a class to be sure that four out of them are born in same month.

3. Attempt any two parts of the following:- **[10x2=20]**

(a) Show that the following pairs of proposition are logically equivalent
(i) $(p \wedge q) \vee (p \wedge \sim q) \equiv p$
(ii) $(p \to q) \wedge (r \to q) \equiv (p \vee r) \to q$

(b) (i) Prove that $\sqrt{3}$ is irrational.
(ii) Show that the proposition $(\wedge \sim q)$ and $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ are contradiction.

(c) (i) Obtain the CNF and DNF of $\sim (p \vee q) \leftrightarrow p \wedge q$.
(ii) Construct the truth table for $(\sim p \to r) \to ((p \to q) \to (p \to r))$

4. Attempt any two parts of the following:- **[10x2=20]**

(a) Define a group. Show that the set of all integers I forms an abelian group with respect to binary operation $*$ defined by the rule

$a * b = a + b + 1$ for all $a, b \in I$.

(b) (i) Prove that every group of order 3 is abelian group.

(ii) Prove that the intersection of two subgroup is a subgroup.

(c) (i) Show that U(8) and U(12) are isomorphic.

(ii) Find the order of all elements of U(18).

5. Attempt any two parts of the following:- **[10x2=20]**

(a) (i) What is the total number of nodes in a full binary tree with 20 leaves?

(ii) Let$X = \{a, b, c\}$. Define $f: X \to X$ such that

$f = \{(a, b), (b, a), (c, c)\}$. Find (i) f^{-1} (ii) f^2 (iii) f^3

(b) (i) Draw the undirected graph G corresponding to adjancency matrix,

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

(ii) Explain Eulerian graph and Hamiltonian graph with their properties and some examples.

(c) (i) Draw graphs representing problems of

(A) Two houses and three utilities

(B) Four houses and four utilities.

(ii) Define the following with one example of each : Binary Trees, Spanning Trees