PAPER CODE: OE-038 Roll No.

B. Tech.

(SEM III) ODD SEMESTER EXAMINATION 2015-16

DISCRETE MATHEMATICS

Time: 3 Hours Max. Marks: 100

Note: Attempt all the questions. All questions carry equal marks.

1. Attempt any **two** parts of the following: **10x2=20**

(a) State and prove hand-shaking lemma. Also, prove that the number of vertices with odd degree in a graph is always ODD.

(b) Define an eulerian graph. Prove that a graph is eulerian if and only if every vertex of the graph is of even degree.

(c) Define a planar graph. Detect the planarity of K_5, $K_{3,3}$, Hamiltonian graph and Petersen graph.

2. Attempt any **two** parts of the following: **10x2=20**

(a) Solve the hollowing recurrence relations:

i. $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial conditions $a_0=2$, $a_1=5$ and $a_2=15$.

ii. $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

(b) A new employee at an exciting new software company starts with a salary of Rs. 12.00 lacs per annum and is promised that at the end of each year her salary will be made 1.25 times her salary of the previous year with an extra increment of Rs. 1.00 lacs for each year she has been with the company. Construct a recurrence relation for her salary for her n^{th} year of employment and solve this recurrence relation to find her salary for her n^{th} year of employment.

(c) Find the number of positive integers not exceeding 10,000 that are not divisible by 3,7 and 11.

3. Attempt any **two** parts of the following: **10x2=20**

(a) Define a cyclic group. Show that every cyclic group is abelian. Verify whether U(20) is a cyclic group.

(b) Find a cyclic subgroup of order 4 in U(40).

(c) Find the order of every element of S_3, the group of permutations of three letters.

4. Attempt any **two** parts of the following: **10x2=20**

(a) Find the examples of relations R_1, R_2, R_3, R_4 and R_5 on a set consist of four elements such that $R_1 o R_2 \neq R_2 o R_1$, $R_3 o R_4 = R_4 o R_3$ and $R_5^2 = R_5 o R_5 = R_5$.

(b) For a set X with n elements find the number of relations on X which are

i. symmetric ii. both reflexive and symmetric

(c) Given the relational matrices M_R and M_S, find M_{RoS}, M_R^{-1}, M_S^{-1}, M_{SoR} and show that $M_{(RoS)}^{-1} = M_{S^{-1} o R^{-1}}$ where

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \qquad M_S = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

5. Attempt any **two** parts of the following: **10x2=20**

(a) Construct truth tables for the following statements : ν <=> $\wedge$

i. $(p \nu q \Rightarrow s) \Leftrightarrow (p \Rightarrow s) \nu (q \Rightarrow s)$

ii. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [(p \Rightarrow r)]$

(b) Define a Hamiltonian graph. Find out five Hamiltonian circuits in the **Hamiltonian** graph

(c) Define a spanning tree of a graph. Find out five spanning trees in each of K_5 and $K_{3,3}$.