Paper Code: MCA 114 **Roll No.**

MCA
(SEM II) EVEN SEMESTER EXAMINATION 2015-16
DISCRETE MATHEMATICS

[Time: 3 hrs.] **[Max. Marks: 100]**

Note- ***Attempt All Questions. All Questions carry equal marks*:-**

1. Attempt *any four* parts of the following: **[5X4 = 20]**

(a) Let S_k={ …..-1, 0, 1,......}. Find $\bigcup_{k=1}^{n} S_k$ and $\bigcup_{k=1}^{\infty} S_k$.

(b) Prove S and T be two sets. Then prove the following:

(i) $S - T \subseteq S$ (ii) $S \cup (T - S) = S \cup T$

(c) State the conditions for which a relation R on a set of S of integers is not reflexive and not transitive.

(d) Let f: R →R, where $f(x) = x^2$. Find $f^{-1}(-\infty \leq x \leq 0)$.

(e) Prove the following inequality using mathematical induction

$1 + n^3 > n + n^2$ for all natural number $n \geq 2$.

(f) Let f: RxR→R x R. Find f^{-1} if exists for $f(x, y) = (x^{1/2}, y^{1/2})$

2. Attempt *any two* parts of the following: **[10X 2 = 20]**

(a) Define Abelian group. Prove that a group G is Abelian if and only if

$(x\, y)^{-1} = x^{-1}\, y^{-1}$ for all x, y in G.

(b) (i) Prove that the intersection of two subgroups of a group *G* is also a subgroup of *G*.

(ii) Let G = $(Z^2, +)$ be a group and let H be a subgroup of G, where H = {(x, y) | x = y}

Find the left cosets of H in G.

(c) (i) Let *(R, +, .)* be a ring. If a^2 =a. a=a , ∀ a ∈ R, then prove that

$a + a = 0$; and $ab + ba = 0$;

(ii) Show that *R* is a commutative ring.

3. Attempt *any two* parts of the following: **[10X 2 = 20]**

(a) What is a Hasse diagram? Consider a set S = {1, 2, 3, 4, 5, 6, 7, 8, 9}. Draw the Hasse diagram when partially ordered by divisibility.

(b) (i) Let a Boolean expression f(x, y z) = x y′ + xyz′ + x′yz′. Then show that

f(x, y, z) + z′ ≠ f(x, y z)

(ii) Let a Boolean function f(x, y, z) = x′y + xyz′, then show that ff′= 0.

(c) (i) Simplify the following Boolean function using Karnaughmap

$F(w, x, y, z) = \prod (0, 1, 2, 3, ,4 ,6, 12)$

(ii) Explain the working a flip-flop.

4. Attempt ***any two*** parts of the following: **[10X 2 = 20]**

(a) Prove the validity of the following argument

$P \rightarrow Q,\ \neg P \rightarrow R$, and $R \rightarrow S \,/\, \therefore \neg Q \rightarrow S$

(b) Determine a suitable conclusion for the given premises

$P \rightarrow (R \wedge S),\ \neg(R \wedge S)$ and $\neg P \rightarrow S$

(c) Compute the relationship between quantifiers. Explain your answer with example using first order predicates.

5. Attempt ***any two*** parts of the following: **[10X 2 = 20]**

(a) (i) What is a Binary tree? Determine a postfix form of the following expression

$\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$

(ii) Define planar graphs and bipartite graphs with examples. Also describe graph coloring problem.

(b) (i) Let $T(n) = 3\,T(n/2) + 5$ and $T(1) = 3$. Find $T(2^k)$, where k is a positive integer.

(iii) Solve the following recurrence

$T(n) = 2\,T(\sqrt{n}) + 1$, where n is a perfect square greater than 1 and $T(2) = 3$.

(c) (i) Let $\Sigma = \{0, 1\}$. Determine the number of strings of length 8 has exactly three 1's.

(ii) Explain Polya's counting theory with an example.