Paper Code: EIT-071	**Roll No.:**
Paper ID:	

B.TECH

(SEM VII) SEMESTER EXAMINATION 2015-16

DISCRETE STRUCTURE

TIME: 3 hrs.] **[Max. Marks: 100]**

Note: ***Attempt All Questions. All Question carry equal marks. Make suitable assumptions wherever necessary.***

Q1. . Attempt any four parts of the following :- **(5 x 4=20)**

(a) Prove that A B ∩ = A∪ B by giving a Venn diagram proof.

(b) Find the inverse of the function f or else explain why the function has no inverse.

(i) $f : R \rightarrow R$ where $f(x) = 3x - 5$. (ii) $f : R \rightarrow R$ where $f(x) = \lfloor 2x \rfloor$.

(c.) If $M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Determine if R is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.

(d) Suppose that R and S are equivalence relations on a set A. Prove that the relation R ∩ S is also an equivalence relation on A.

(e) Use mathematical induction to show that $3^n < n!$ Whenever n is an integer with $n \geq 7$.

(f) What is a recursive algorithm? Describe a recursive algorithm for computing the greatest common divisor of two positive integers.

Q2. . Attempt any two parts of the following :- **(5x 4=20)**

(a) Define identity and zero elements of a set under a binary operation *. What do you mean by an inverse element? Find the inverse of an element if binary operation * over set of integers(I) is defined by

$a * b = a + b + 1$ for all a, b : I

(b) Define the terms semigroup and monoid. Is (I, *) smigroup? Is (I, *) monoid? Where I is the set of integers and binary operation * as in Q. 2(a).

(c) Let S = {1, 3, 7, 9}. Show that set S along with multiplication mod 10 form a group.

(d) Let S = {1, -1, i, -i}, i = $\sqrt{-1}$, and G = (S, complex number multiplication). Show that H = {1, -1} is a subgroup of G. Determine the all left cosets of H.

(e) How symmetric groups are defined? Explain with an example.

(f) Give the definition of Ring, Integral domain and Field.

3. **Attempt any two parts of the following :-** **(10x2=20)**

(a) Define a partial ordering. Show that divisibility relation on the set of positive integers is a partial order. Draw the Hasse diagram of the divisibility relation on the set {2, 3, 5, 9, 12, 15, 18}.

(b) Define a lattice. Give an example of a poset with five elements that is a lattice and an example of a poset with five elements that is not lattice.

(c) Use a Karnaugh map to minimize the sum-of-products of the following expression and draw the logic diagram of resultant expression.

$$xyz + x y'z + x' y'z + x y'z' + x'yz + x' y'z'.$$

4. **Attempt any two parts of the followings:-** **(10x2=20)**

(a) (i) Write the truth table for the proposition $\neg(r \to \neg q) \vee (p \wedge \neg r)$.
(ii) What does it mean for two propositions to be logically equivalent? Show in at least two different ways that the compound propositions $\neg p \vee (r \to \neg q)$ and $\neg p \vee \neg q \vee \neg r$ are equivalent.

(b) (i) Prove that $((p \to \neg q) \wedge q) \to \neg p$ is a tautology using propositional equivalence and the laws of logic.
(ii) Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.

(c) (i) Suppose the variable *x* represents students and *y* represents courses, and:
$U(y)$: *y* is an upper-level course $M(y)$: *y* is a math course $F(x)$: *x* is a freshman
$A(x)$: *x* is a part-time student $T(x,y)$: student *x* is taking course *y*.
Write the statement using these predicates and any needed quantifiers.

(1) Every student is taking at least one course
(2) There is a part-time student who is not taking any math course.
(3) Every part-time freshman is taking some upper-level course.

(ii) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will held and the lifesaving demonstration will go on," "If the sailing race is held, then trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

5. **Attempt any two parts of the followings:-** **(10x2=20)**

(a) Define preorder, inorder, and postorder tree traversal. Give an example of preorder, postorder, and inorder traversal of a binary tree of your choice with at least 12 vertices.

(b) Define a linear homogeneous recurrence relation of degree k. Find an explicit formula for the Fibonacci numbers.

(c) Write short notes on the following.
(i) Planar Graphs.
(ii) Pigeon hole principle.