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Paper Code: EAS-203	Roll No.										

B.Tech.
(SEM II) BACK PAPER EXAMINATION, 2015-16
ENGINEERING MATHEMATICS-II

[Time: 3 hrs.] **[Max. Marks: 100]**

Note- ***Attempt All Questions. All Questions carry equal marks*:-**

$\mathbf{Q1}.Attempt\ any\ \mathbf{Two}\ parts\ of\ the\ following.$ $(10X2 = 20\ Marks)$

$(a)\ Solve\ \frac{d^3y}{dx^3} + y = \sin 3x - cos^2\frac{x}{2}.$

$(b)\ Solve\ the\ simultaneous\ equations$

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$

$(c) An\ uncharged\ condesnser\ of\ capacity\ C\ is\ charged\ by\ applying\ an\ e.m.f.\ E\sin\frac{t}{\sqrt{LC}}$ $through\ leads\ of\ self-inductance\ L\ and\ negligable\ resistance.$ $Prove\ that\ at\ time\ t, the\ charge\ on\ one\ of\ the\ plates\ is$ $\frac{EC}{2}\left[sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}cos\frac{t}{\sqrt{LC}}\right]$

$\mathbf{Q2}.Attempt\ any\ \mathbf{Two}\ parts\ of\ the\ following.$ $(10X2 = 20\ Marks)$

$(a).Find\ series\ solution\ about\ x = 0\ of\ the\ differential\ equation$

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - \left(x^2 + \frac{5}{4}\right)y = 0.$$

$(b).Express\ J_4(x) in\ the\ term\ of\ J_0(x) and\ J_1(x).$

$(c).Prove\ that\ \int_{-1}^{+1} P_m(x)P_n(x)dx = 0, if\ m \neq n\ for\ Legendre's\ polynomials.$

$\mathbf{Q3}.Attempt\ any\ \mathbf{Two}\ parts\ of\ the\ following.$ $(10X2 = 20\ Marks)$

$(a).A\ periodic\ function\ of\ period \frac{2\pi}{\omega} is\ definded\ by$

$$f(t) = \begin{cases} \text{E}\sin\omega t & for\ 0 \le t \le \frac{\pi}{\omega} \\ 0 & for \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$$

$Where\ E\ and\ \omega\ are\ positive\ constants. Find\ the\ Laplace\ transform\ of\ the\ function.$

$(b).Using\ Convolutiion\ theorem, find\ L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}.$

$(c).Solve\ \frac{d^2y}{dt^2} + y = t\cos 2t,\ give\ y = 0 = \frac{dy}{dt}\ for\ t = 0, by\ using\ Laplace\ transform.$

Q4. *Attempt any* **Two** *parts of the following.* (10X2 = 20 Marks)

(a). Find the Fourier series for the functions f defined by $f(x) = x - x^2, -\pi < x < \pi$.

Deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

(b). Determine the Fourier series for sawtooth function f defined by

$$f(t) = t, \quad t \in (-\pi, \pi)$$

$$f(t) = f(t + 2\pi)$$

(c). Solve $(D^3 - 3D\,D'^2 + 2D'^3)z = \cos(x + 2y) - e^y\,(3 + 2x)$

where symbols have their usual meaning.

Q5. *Attempt any* **Two** *parts of the following.* (10X2 = 20 Marks)

(a). Solve by sepration of variables method

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ if } u(x,0) = \begin{cases} 2x, & \text{when } 0 \le x \le l/2 \\ 2(l - x), & \text{when } \frac{l}{2} \le x \le l \end{cases}$$

(b). If a string of lenght l is initially at rest in the equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}$, find the displacement $y(x,t)$.

(c). Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0,y) = u(l,y) = u(x,0) = 0, u(x,a) = \sin\frac{n\pi x}{l}$.