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CS-303

Roll No. | | | | | | | | | | |

B.Tech.
(SEM III) ODD SEMESTER THEORY EXAMINATION, 2015-16

COMPTER BASED NUMERICAL AND STATISTICAL TECHNIQUES

Time: 2 Hours **Maximum Marks:** 50

Note:
(i) *Attempt **ALL** questions.*
(ii) *Notations/ Symbols/ Abbreviations used have usual meaning.*

Q.1. ***Attempt any FOUR parts of the following.*** **3.5x4=14**

a) Explain the following terms.
Machine Epsilon, Inherent error, Truncation error

b) Design a suitable example to show that associative law of addition may not hold in numerical computations.

c) Write an algorithm for finding real root of an equation using Secant method.

d) The equation
$$x^3 - 5x^2 + 4x - 3 = 0$$
has one root near **4** which is to be computed by iterative method defined as below.
$$x_0 = 4$$
$$x_{n+1} = (3 + (k-4)\,x_n + 5x_n^2 - x_n^3) / k \text{ ; where } k \text{ is integer.}$$
Determine the value of **k** which will give fastest possible rate of convergence.

e) Perform two iterations of the Birge-vieta method to find a real root correct to three decimal places of the equation:
$$x^6 - x^4 - x^3 - 1 = 0.$$
Use initial approximation **p=1.5**

f) State Sturm theorem. Illustrate how Sturm theorem can be used to obtain the exact number of real roots of the given polynomial between 0 and 3. Also determine the multiplicity of the roots.
$$x^3 + x^2 - x - 1 = 0$$

Q.2. ***Attempt any TWO parts of the following:*** **6x2=12**

a) Obtain the natural cubic spline interpolating polynomial valid in the interval [3, 4] for the following function.

x_i	1	2	3	4
$f(x_i)$	3	10	29	65

b) Using Gram Schmidt orthogonalization process, compute the first three orthogonal polynomials for the interval **[0,2]** with respect to weight function **W(x)=1.** Use these polynomials to obtain normal equations for second degree approximation of $\mathbf{f(x) = x^3}$ on **[0,2]** according to least squares principle.

c)

i) Prove the following relation between forward difference operator and backward difference operator.

$$\Delta + \nabla = \Delta / \nabla - \nabla / \Delta$$

ii) What are Chebyshev polynomials? Find the best uniform approximation of degree 2 to $\mathbf{5x^3 + 3x^2 - 1}$ on **[-1, 1]** using Lanczos economization.

Q.3. ***Attempt any TWO parts of the following:*** **6x2=12**

a) Determine the value of $\lambda_1, \lambda_2, \lambda_3, x_1$ so that the following formula for computing the integral is exact for polynomials of the degree as high as possible.

$$\int_{-1}^{1} (a-x) f(x)\, dx = \lambda_1 f(-x_1) + \lambda_2 f(0) + \lambda_3 f(x_1)$$

b)

i) Derive the relation between differential operator $D \equiv d/dx$ and the forward difference operator Δ.

ii) Use Trepezoidal rule with four subintervals to evaluate the following integral correct to three decimal places.

$$I = \int_{1}^{5} \int_{1}^{5} (dx / (x^2 + y^2)^{1/2}) dy$$

c) Evaluate the following integral using Guass-Legendre 3-point method.

$$I = \int_{1}^{2} dx / (1+x^3)$$

Q.4. ***Attempt any TWO parts of the following:*** **6x2=12**

a) Set up the Successive Over Relaxation (SOR) iteration scheme in matrix form for solving the given system of simultaneous using relaxation parameter $\omega = 1.132$ for equations.

$$\begin{aligned} 4x \quad\quad + 2z &= 4 \\ 5y + 2z &= -3 \\ 5x + 4y + 10z &= 2 \end{aligned}$$

b) Use classical Runge-Kutta fourth order method with step size **h = 0.2** to find numerical solution at x = 0.8 for

$$dy/dx = (x+y)^{1/2} ; \; y(0.4) = 41$$

c)

(i) What do you understand by ill-conditioned system of equations? Obtain the condition number of following system of equations using spectral norm.

$$\begin{aligned} 2.1x + 1.8y &= 2 \\ 6.2x + 5.3y &= 6 \end{aligned}$$

(ii) Explain the following terms.
Type-I Error, Type-II Error, Level of significance