Paper Code: CS-302	Roll No.:
Paper ID:	

B.TECH
(SEM III) SEMESTER EXAMINATION 2015-16
DISCRETE STRUCTURE AND GRAPH THEORY

[TIME: 3 hrs.] **[Max. Marks: 100]**

Note: ***Attempt All Questions. All Question carry equal marks. Make suitable assumptions wherever necessary.***

Q1. . Attempt any four parts of the following :- **(5 x 4=20)**

(a) Let A, B, and C be sets. Prove or disprove that $A - (B \cap C) = (A - B) \cup (A - C)$.

(b) Let X = {a, b, c}. Define $f : X \to X$ such that

f = { (a, b), (b, a), (c, c) }.

Find f^{-1}, f^2, and f^3

(c.) If $M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Determine if R is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.

(d) Suppose A is the set composed of all ordered pairs of positive integers. Let R be the relation defined on A where (a, b) R (c ,d) means that a + d = b + c.

(i) Prove that R is an equivalence relation.

(ii) Find [(2,4)].

(e) Use the Principle of Mathematical Induction to prove that $1 + 2^n \le 3^n$ for all $n \ge 1$.

(f) Explain why a function is well defined if it is defined recursively by specifying $f(1)$ and a rule for finding $f(n)$ from $f(n - 1)$.

Q2. . Attempt any four parts of the following :- **(5x 4=20)**

(a) Define semi-group and monoid. Let Q be the set of rational numbers and operation * is defined as $a * b = a + b - ab$ for $a, b \in Q$.

Is (Q, *) a semi-group or monoid? Justify your answer.

(b) For a group G prove the following
   (i) Each element a in G has only one inverse.
   (ii) If a and b are elements of group G, then $(a\ b)^{-1} = b^{-1}\ a^{-1}$.

(c) Let S = {1, 3, 7, 9} and G = (S, multiplication mod 10). Determine all left cosets and right cosects of the subgroup {1, 9}.

(d) Discuss a symmetric group of order 6 and degree 3.

(e) Let G be the group of integers under the operation of addition and G' be the group of all even integers under the operation of addition. Show that the function f: G → G' defined by $f(a) = 2a$ is an isomorphism.

3. **Attempt any two parts of the following :-** **(10x2=20)**

(a) (i) Determine whether the given relation is a partial order. Explain your answer.

(A) S be any set; a R b if and only if a = b.

(B) S be the set of parallel lines in the plane; l_1 R l_2 if and only if l_1 coincides with l_2 or l_1 is parallel to l_2.

(iii) If (A, ≤) and (B, ≤) are posets, then show that (A x B, ≤) is a poset, with partial order defined by

(a, b) ≤ (a', b') if a ≤ a' in A and b ≤ b' in B.

(b) Define a Lattice. Show that D(40), set of all positive divisors of 40, is a lattice.

(c) (i) Show that the Boolean function F given by F (x y z) = x (z+ yz) + y ((xz)'x)' simplifies to xz + x'y , by using only the definition of a Boolean algebra.

(ii) Describe the Boolean duality principle. Write the dual of Boolean function x + x y = x + y

4. **Attempt any two parts of the followings:-** **(10x2=20)**

(a) (i) Write the truth table for the proposition $\neg(r \rightarrow \neg q) \vee (p \wedge \neg r)$.

(ii) Find a proposition with three variables *p*, *q*, and *r* that is true when exactly one of the three variables is true, and false otherwise

(b) (i) Prove that $(q \wedge (p \rightarrow \neg q)) \rightarrow \neg p$ is a tautology using propositional equivalence and the laws of logic.

(ii) Write the contrapositive, converse, and inverse of the following:

If you try hard, then you will win.

(c) (i) Consider the statement "Given any positive integer, there is a greater positive integer." Symbolize this statement with and without using the set of positive integers as the universe of discourse.

(ii) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will held and the lifesaving demonstration will go on," "If the sailing race is held, then trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

5. **Attempt any two parts of the followings:-** **(10x2=20)**

(a) (i) Determine if the relation R = {(1, 7), (2, 3), (4, 1), (2, 6), (4, 5), (5, 3), (4, 2)} is a tree on the set A = {1, 2, 3, 4, 5, 6, 7}. If it is a tree, what is the root? If it is not a tree, make the least number of changes necessary to make a tree and give the root.

(ii) Create a binary tree for which the results of performing a preorder search are $S_1TRE_1S_2S_3E_2D$ and for which the results of performing a post order search are $DE_2S_3S_2E_1RTS_1$. Assume that to visit a vertex means to print the contents of the vertex.

(b) What do you mean by a recurrence? Find a formula for the n^{th} term of the Fibonacci sequence.

(c) Write short notes on the following.

(i) Planar Graphs.

(ii) Homomorphism of graphs

(ii) Pigeon hole principle.