

Paper Code: AS-201

Roll No.

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B.Tech.

**(SEM II) EVEN SEMESTER EXAMINATION, 2015-16  
ENGINEERING MATHEMATICS-II**

[Time: 3 hrs.]

[Max. Marks: 100]

Note- Attempt All Questions. All Questions carry equal marks:-

Q1. Attempt any **Two** parts of the following.

(10X2 = 20 Marks)

(a) Solve  $\frac{d^3y}{dx^3} + y = \sin 3x - \cos^2 \frac{x}{2}$ .

(b) Solve the simultaneous equations

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2 \cos t - 7 \sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4 \cos t - 3 \sin t$$

(c) An uncharged condenser of capacity  $C$  is charged by applying an e. m. f.  $E \sin \frac{t}{\sqrt{LC}}$  through leads of self-inductance  $L$  and negligible resistance.Prove that at time  $t$ , the charge on one of the plates is  $\frac{EC}{2} \left[ \sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right]$ Q2. Attempt any **Two** parts of the following.

(10X2 = 20 Marks)

(a) Find series solution about  $x = 0$  of the differential equation

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - \left(x^2 + \frac{5}{4}\right)y = 0.$$

(b) Express  $J_4(x)$  in the term of  $J_0(x)$  and  $J_1(x)$ .(c) Prove that  $\int_{-1}^{+1} P_m(x)P_n(x) dx = 0$ , if  $m \neq n$  for Legendre's polynomials.Q3. Attempt any **Two** parts of the following.

(10X2 = 20 Marks)

(a) A periodic function of period  $\frac{2\pi}{\omega}$  is defined by

$$f(t) = \begin{cases} E \sin \omega t & \text{for } 0 \leq t \leq \frac{\pi}{\omega} \\ 0 & \text{for } \frac{\pi}{\omega} \leq t \leq \frac{2\pi}{\omega} \end{cases}$$

Where  $E$  and  $\omega$  are positive constants. Find the Laplace transform of the function.(b) Using Convolution theorem, find  $L^{-1} \left\{ \frac{1}{s^2(s+1)^2} \right\}$ .(c) Solve  $\frac{d^2y}{dt^2} + y = t \cos 2t$ , give  $y = 0 = \frac{dy}{dt}$  for  $t = 0$ , by using Laplace transform.

**Q4.** Attempt any **Two** parts of the following. (10X2 = 20 Marks)

(a). Find the Fourier series for the functions  $f$  defined by  $f(x) = x - x^2, -\pi < x < \pi$ .

Deduce that  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$ .

(b). Determine the Fourier series for sawtooth function  $f$  defined by

$$f(t) = t, \quad t \in (-\pi, \pi)$$

$$f(t) = f(t + 2\pi)$$

(c). Solve  $(D^3 - 3D D'^2 + 2D'^3)z = \cos(x + 2y) - e^y (3 + 2x)$

where symbols have their usual meaning.

**Q5.** Attempt any **Two** parts of the following. (10X2 = 20 Marks)

(a). Solve by separation of variables method

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ if } u(x, 0) = \begin{cases} 2x, & \text{when } 0 \leq x \leq l/2 \\ 2(l-x), & \text{when } \frac{l}{2} \leq x \leq l \end{cases}$$

(b). If a string of length  $l$  is initially at rest in the equilibrium position and each of its points is given the velocity  $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}$ , find the displacement  $y(x, t)$ .

(c). Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  subject to the conditions  $u(0, y) = u(l, y) = u(x, 0) = 0, u(x, a) = \sin \frac{\pi x}{l}$ .