Paper Code: AS 301	Roll No.									

B.Tech.
(SEM III) ODD SEMESTER EXAMINATION 2015-16
MATHEMATICS III

[Time: 3 hrs.] **[Max. Marks: 100]**

Note- ***Attempt All Questions. All Questions carry equal marks*:-**

Q1 Attempt any **four** parts of the following. **(5x4=20)**

(a) $In\ a\ two-dimensional\ fluid\ flow, the\ stream\ function\ \Psi\ is\ given\ by\ \Psi = \tan^{-1}\frac{y}{x}$. $Find\ the\ velocity\ potential.$

(b) $Write\ necessary\ and\ sufficient\ conditions\ for\ f(z)\ to\ be\ analytic. Obtain\ polar\ form\ of\ Cauchy-Riemann\ equations.$

(c) $Find\ the\ residue\ at\ the\ singular\ point\ of\ the\ function\ z^{-5}\cos z.$

(d) $Evaluate\ \int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, $where\ c\ is\ the\ circle\ |z| = 3.$

(e)Apply Calculus of residues to evaluate $\int_0^\infty \frac{x \sin ax}{x^2 + k^2}\ dx \quad , \quad a > 0\ , k > 0.$

(f). $Find\ the\ expansion\ of\ the\ function\ f(z) = \frac{1}{(z^2+1)(z^2+2)}\ in\ powers\ of\ z\ when\ (i)|z| < 1 (ii) 1 < |z| < \sqrt{2}$.

Q2. Attempt any **two** parts of the following. **(10x2=20)**

(a) $Define\ Fourier\ Sine\ transform\ and\ its\ inversion. Find\ f(x)\ if\ its\ sine\ transform\ is\ \frac{e^{-ap}}{p}$.

$Hence\ deduce\ f_s^{-1}\left\{\frac{1}{p}\right\}.$

(b) $Solve\ the\ integral\ equation.$

$$\int_0^\infty f(t)\cos \omega t\ dt = \begin{cases} 1-\omega & for\ 0 \le \omega \le 1 \\ 0 & for\ \omega > 1. \end{cases}$$

$$Hence\ evaluate \int_0^\infty \frac{\sin^2 t}{t^2}\ dt.$$

(c)*The economy of a country basically depends on the following three factors :*

(i) *national income,* (ii) *consumption, and* (iii) *investment. These can be expressed by the difference equation with constant coefficient. Solve the difference equation using* $z-$ *transform*

Q3. Attempt any **two** parts of the following. **(10x2=20)**

(a) *Solve the following system of equations by Gauss – Seidel method correct to four decimal places*:

$x+y+54z=110;$ $27x+6y-z=85;$ $6x+15y+2z=72$

(b)*A rocket is launched vertically upward from the ground. Its acceleration is registered during the first* 80 *seconds and is given in the following table*:

$t\ (in\ sec)$	0	10	20	30	40	50	60	70	80
$a\ (in\ m/sec^2)$	30.00	31.63	33.44	35.47	37.75	40.43	43.29	46.69	50.67

Find the velocity and the height of the rocket at $t=80$ *seconds*

by Trapezoidal rule and Simpson's $\frac{1}{3}$ *rule respectively .*

(c) *Discuss Picard's method of successive Integration. Using Picard's method, solve*

$\frac{dy}{dx}=x^2-y,\ y(0)=1\ \ for\ x=0.2$

Q4. Attempt any **two** parts of the following. **(10x2=20)**

(a) *Derive the Newton* $-$ *Raphson's formula for finding root of a function. Using this formula find a real root of the equation* $x^3-3x-5=0$. *Correct upto four decimal palces.*

(b)*The viscosity of a certain kind of oil is experimently measured at different temperatures as shown in the following table*:

Temperature in ℃	110	130	160	190
Viscosity of the oil	10.8	8.1	5.5	4.8

Using this table, find the viscosity of this oil at $140°C$, *by Lagrange's method of interpolation.*

(c)*Define interpolation and extrapolation give brief account of its applications in Engineering .*

Estimate the number of students who obtained marks between 40 *and* 45

from the following table:

Marks :	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

Q5. Attempt any **two** parts of the following. **(10x2=20)**

(a) *A study of price of a certain commodity at Gorakhpur and Lucknow yields the following data*:

	Gorakhpur (Rs)	*Lucknow* (Rs)
Average price /Kilo	2.463	2.797
Standard deviation	0.326	0.207

'r' between prices at Gorakhpur and Lucknow is 0.774. *Estimate, from the above data, most likely price at Gorakhpur corresponding to the price of Rs* 3.052 *per kilo at Lucknow.*

(b) *Fit a Parabola to the following* :

x	1	2	3	4	5	6	7	8	9
x	2	6	7	8	10	11	11	10	9

(c) *Give brief idea and application of Spearman's rank correlation coefficient.*

Calculate Spearman's rank correlation for the following data , and interpret the result.

Marks in Mathematics	15	18	21	24	27	30	36	39	42	48
Marks in Statistics	25	25	27	27	31	33	35	41	41	45