AS-101 **Roll No.** | | | | | | | | | | |

B.Tech.
(SEM-I) ODD SEMESTER EXAMINATION 2015-16
MATHEMATICS-I

Time 3 Hours **Maximum Marks 100**

Note: Attempt all questions. All questions carry equal marks.

Q.1.Attempt any **Two** parts of the following. **(2 x 10=20)**

(a) If $y = \sin(a \sin^{-1} x)$, then prove that (i) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (a^2 - n^2)y_n = 0$ and (ii) find $(y_n)_0$

(b) Verify Euler's theorm for (i) $u = x^2yz - 4y^2z^2 + 2xz^3$ (ii) $u = \frac{x+y+z}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$

(c) (i) Find the value of $\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}}$ if $u = x^2 + y^2 + z^2 - 2xyz = 1$

(ii) Find $\frac{dz}{dt}$ if $z = \sin^{-1}(x-y), x = 3t, y = 4t^3$

Q.2.Attempt any **Two** parts of the following. **(2 x 10=20)**

(a)(i) If $x = r\sin\theta\cos\emptyset$, $y = r\sin\theta\sin\emptyset$, $z = r\cos\theta$, then show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta$.

(ii) Find the shortest and longest distance from the point $(1,2,-1)$ to the Sphere $x^2 + y^2 + z^2 = 24$.

(b) Trace the curve (i) $r^2 = a^2\cos 2\theta$ (ii) $x^3 + y^3 = 3axy$.

(c)(i) Evaluate $\sqrt{25.15}$ using Taylor's theorem (ii) Expand $e^{ax}\sin by$, into powers of x & y upto third degree terms,

Q.3.Attempt any **Two** parts of the following. **(2 x 10=20)**

(a) Find the non-singular matrices P and Q such that the normal form of A is PAQ.

Where $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$, Also find the rank of A.

(b) (i) Find the eigenvalues and corresponding eigenvectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

(ii) For what values of A and B the matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as its eigen values .

(c) Verify Cayley Hamilton theorem for matrix $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$ and hence find A^{-1}

Q.4.Attempt any **Two** parts of the following. **(2 x 10=20)**

(a) (i) Evaluate $\iint_R (x+y)\,dxdy$ where R is the region bounded by $y = 0, x + y = 2$ and $y^2 = x$.

(ii) Evaluate $\iiint_R (x+y+z)\,dxdydz$ where $R: (0 \le x \le 1), (1 \le y \le 2), (2 \le z \le 3)$

(b) Change the order of integration for $I = \int_{x=0}^{2} \int_{y=x^2/4}^{3-x} xy \; dydx$ and hence evaluate it.

(c)(i) Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$, $(p > 0, q > 0)$

(ii)Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$ using Beta and Gamma function.

Q.5.Attempt any **Two** parts of the following. **(2 x 10=20)**

(a) (i)Define gradient ,divergence and curl of a vector and the physical interpretation of curl of a vector.

(ii)Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

(b) (i) If $\vec{a}$ is a constant vector ,evaluate div $(\vec{r} \times \vec{a})$ and curl $(\vec{r} \times \vec{a})$ where $\vec{r}$ is a position vector

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

(ii)Evaluate curl (grad ϕ) where ϕ is a scalar.

(c) Verify Green's theorem in the plane for $\int_c [(3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy]$ where C

is the region bounded by parabolas $y^2 = x$ and $y = x^2$.

OR

Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\,\hat{\jmath}$ in the rectangular region bounded by

$x = 0, x = a, y = 0$ and $y = b$